A Model for Measurements of Lognormally Distributed Environmental Contaminants

Charles B. Davis, PhD
EnviroStat

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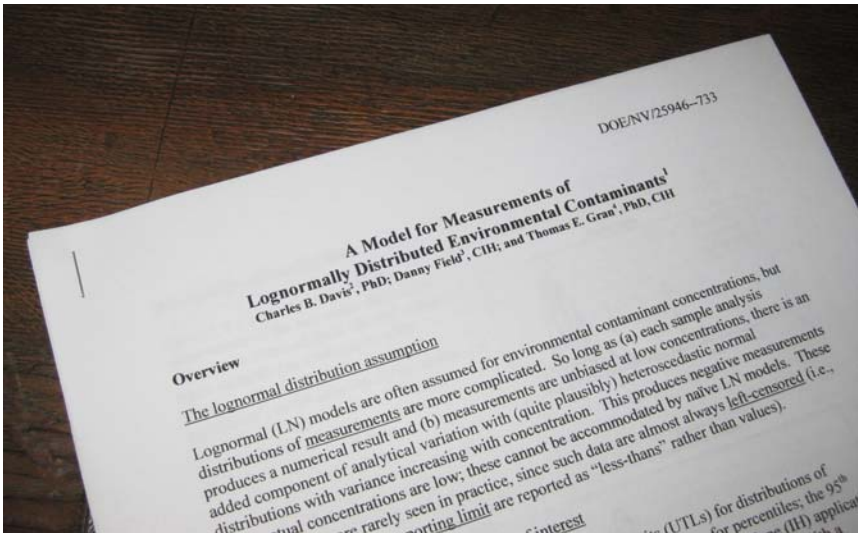
This Study

- **This work motivated by Be surface contamination surveys at DOE facilities associated with the NTS**
- **Collaborators and sponsors**
  - Charles Davis, former Math Professor, now Principal Statistician of EnviroStat
  - Dan Field, now Chief Industrial Hygienist for the DOE National Nuclear Security administration in Washington, DC
  - Tom Gran, IH Manager for National Security Technologies at NTS
- **The Paper: same title, 3 authors, 27 pages, request pdf by email**
  - **charles.davis@envirostat-nv.com**
  - Will present a few highlights/surprises here today

DMM | BHSC, November 5, 2009 | Slide 2 of 30

During the NTS beryllium facility surveys well over 12,000 wipe samples were collected, mostly analyzed by ICP-AES with a few ICP-MS included as well. Nearly all data are uncensored ("raw"); there are very few "less-thans". This, along with some of the contingencies that arose along the way, provided an in-depth look at the actual statistical properties of these mostly low-level data. Such an in-depth look is rare; nearly always the details of such data are hidden below a Reporting Limit and therefore not available for study.

The results of this study are in The Paper; the presentation gives only a small glimpse into the results. The Paper can be obtained by email request; it will also be posted on the BHSC website.



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A Model for Measurements of
Lognormally Distributed Environmental Contaminants[1]

Charles B. Davis[2], PhD; Danny Field[3], CIH; and Thomas E. Gran[4], PhD, CIH

Overview

The lognormal distribution assumption

Lognormal (LN) models are often assumed for environmental contaminant concentrations, but distributions of measurements are more complicated. So long as (a) each sample analysis produces a numerical result and (b) measurements are unbiased at low concentrations, there is an added component of analytical variation with (quite plausibly) heteroscedastic normal distributions with variance increasing with concentration. This produces negative measurements ...ual concentrations are low; these cannot be accommodated by naïve LN models. These ...s rarely seen in practice, since such data are almost always left-censored (i.e., ...orting limit are reported as "less-thans" rather than values).

...interest

... (UTLs) for distributions of ... for percentiles; the 95th ... (IH) applica...

Topics

- **Highlights of Aberdeen BHSC Presentation (April 2008)**
  - The DOE Beryllium Facility Survey Problem
    - Upper Tolerance Limits (UTL) for decision-making for facility surveys
  - Statistical nature of ICP-AES and ICP-MS data
    - Many negative measurements!
  - Distribution models for Be data
    - Lognormal (LN) is the "conventional wisdom"
      - Does not accommodate negative values
  - DMM ("Davis Mixed Model")
    - Describes actual data nicely using 4 parameters
    - **Don't have a "gold standard" UTL procedure**
  - DMM very useful for testing candidate procedures

A longer presentation on some of these ideas was given at at the Aberdeen BHSC meeting a year and a half ago. Some material is repeated from that presentation to set the stage for the more recent discoveries. A very important observation is that with real low-level data up to half the data values (sometimes even more) will be negative with ICP-AES analyses.

The UTL approach to decision-making is described on a subsequent slide, as is the DMM. A "gold standard" UTL procedure using the DMM directly is not available; however one can fit a DMM to real data and use it to generate simulation samples with the same statistical properties as real data from the NTS, and use those simulation samples to test candidate UTL (or other) statistical procedures.

Topics, continued

- **Recent insights concerning the DMM and its parameters**
  - δ is virtually irrelevant with a decent lab
- **How well does the three-parameter LN approximation work?**
  - LN3 is sub-model of DMM
  - Have useable UTL procedure
    - Subtleties and caveats
    - **Can be abused by unscrupulous statisticians**
- **What about censored-data LN procedures?**
  - Can work decently if reporting limit (RL) not too high OR TOO LOW
  - Suggestion: trust the model, ignore percent nondetects in your data
    - **Research and political challenge: incorporate these considerations into setting RLs!**
- **Can I use simple nonparametric UTL but get credit for the distance from RL to the Release Criterion (RC)?**
  - The "Quasi-Nonparametric" idea

Items in red are the surprises emerging in the study. Items in green are research and/or implementation challenges based on these results.

One interesting implication of these results is that the performance of some statistical procedures used in facility surveys is sensitive to the RL used, which in turn suggests that one really ought to take the survey objectives into consideration when determining study-specific RLs. Current RLs (and other "limits") are evaluated only from the point of view of interpreting individual measurements. Just how to do this presents both research and political challenges.

DOE Facility Surveys for Beryllium

- **Facility Survey Objective**
  - Determine that a facility is “safe” for release or use; want to protect “all workers” from “exposure” in DOE beryllium surveys (10 CFR 850)
  - Primary exposure route is air, but for facility surveys the emphasis is on surface concentrations
- **Common Industrial Hygiene (IH) Implementation**
  - Obtain swipe samples via appropriate sampling plan; analyze for “total” Be
  - Compare 95%-95% Upper Tolerance Limit (UTL) with 10 CFR 850 Release Criterion (RC) = 0.2 μg/100 cm^2

Upper Tolerance Limits

- **UTL is Upper Confidence Limit (95% level) for 95th Percentile of Distribution of <u>Measurements</u>**
- **Comparing UTL with RC implements Hypothesis Test**
  - H_0: 95th percentile $\geq$ RC ("not safe")
  - H_A: 95th percentile < RC ("safe")
  - with 5% significance level (= 100% - confidence)

The basic idea here is that there is really a statistical hypothesis test lurking here. Using the UTL is a conventional IH way to implement a "safe" vs "not safe" decision regarding removable surface Be concentrations. The null hypothesis is "not safe"; the goal of a survey is to reject that null hypothesis.

In actual surveys there is another decision criterion, which is that any individual wipe measurement whose value exceeds the RC should be investigated. That additional criterion is not evaluated in this presentation or in The Paper.

Upper Tolerance Limits, continued

- **Types of UTLs**
  - Nonparametric (NPUTL)
    - Largest of 59-92 observations, second of 93-123 observations, etc.
  - Parametric, complete data
    - For normal (and lognormal) distributions, formulas of form (*X*-bar) + K S
      - **Using, or even receiving, uncensored chemical data is controversial!**
  - Parametric, censored data
    - Have results, assuming LN model
    - Can handle up to ~ 70% "nondetects"
  - Quasi-nonparametric
    - Not standard

The ONLY UTL procedure that is completely scientifically sound is the nonparametric UTL. Unfortunately, that requires at least 59 wipes for each facility (or survey unit, if facilities are sub-divided or aggregated). That can be expensive and wasteful for smaller and/or cleaner facilities.

"Complete-data" procedures are those that do not allow "less-thans"; "censored-data" procedures do allow less-thans. Statisticians have been developing both complete-data and censored-data UTL and other procedures. Prior to the DMM these have generally been based on lognormal distribution models, which do not allow for the negative values that are ubiquitous in measurements from clean facilities. Censored-data procedures work reasonably well with up to 70 percent nondetects (less-thans) so long as the lognormal model is correct, which is simplistic. One goal of this study is to evaluate how those procedures work under the much more appropriate DMM.

Recently several authors have been suggesting using less heavy-tailed gamma models rather than lognormal models for environmental data; see, for example, the most recent versions of the U.S. EPA's ProUCL software. Empirical evidence that the gamma models are more appropriate for environmental contaminant distributions is very difficult to obtain. The gamma models do seem to provide lower confidence limits UTLs, though, suggesting that the lognormal (LN) models may be conservative.

One recalls that using, or even receiving, uncensored data can be controversial, and requires balancing scientific needs with public relations considerations; see the discussion in Chapter 8, "Data Use, Quality, Reporting, and Communication" in *Beryllium: Environmental Analysis and Monitoring* (M.J. Brisson and A.A. Ekechukwu, editors., Royal Society of Chemistry, 2009).

Statistical Properties of ICP-AES and ICP-MS Measurements

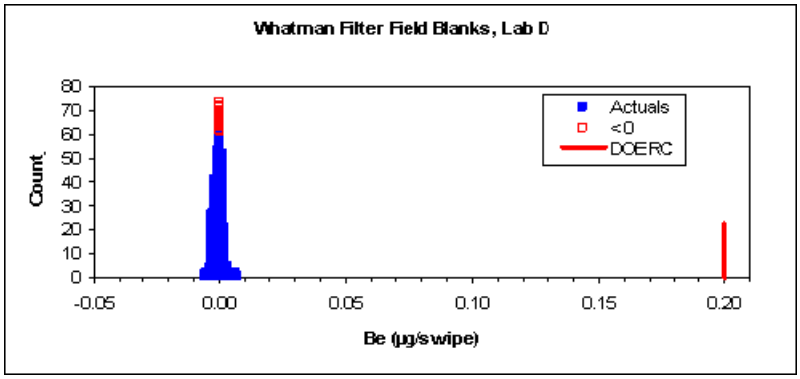
- **Every analysis gives a number!**
- **At fixed concentration (C) distributions are Normal**
  - Standard deviation increases with concentration
  - Variation inherent in sample preparation and laboratory analysis
- **Distribution of concentrations within facility**
  - **NOT OBSERVABLE**
  - DMM assumes LN
    - Probably conservative

The DMM is constructed from two components: the distribution of measurement values at given concentrations, and the distribution of concentrations within a facility. From spiked-sample studies we have a good handle on the former; at given concentrations distributions of measurements resemble normal (Gaussian, bell-shaped) distributions, with standard deviation increasing as concentration increases. This variation is inherent in sample preparation as well as the functioning of the laboratory instruments.

The distribution of actual concentrations within a facility is <u>not observable</u>; we never see concentrations, we see measurements. The LN assumption is used in the DMM; this is probably conservative, as noted previously.

Distributions of Real Facility Data

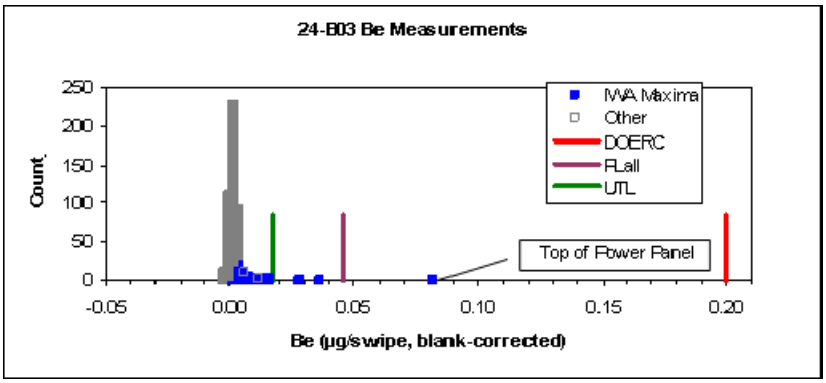
- **Field Blanks, ICP-AES**



Here are a few of the plots of actual data from the Aberdeen presentation. First is a histogram of field blank results (Whatman filter, ICP-AES). The mean is just a little negative; the distribution is virtually symmetric. The RC is included in the plot for reference.

Distributions of Real Facility Data, continued

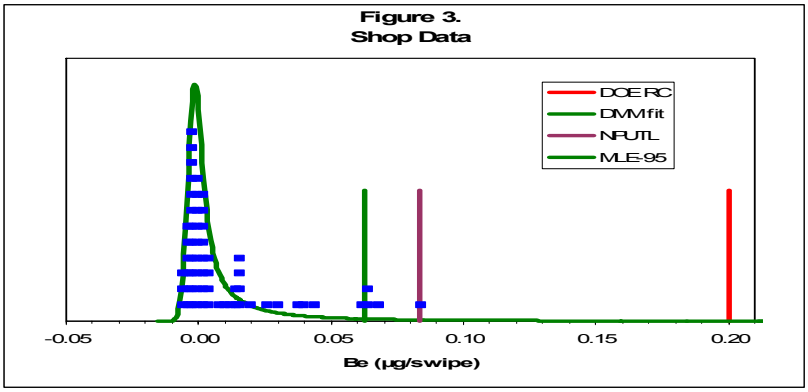
- **North Las Vegas B-3**
  - Sampled while still occupied before evacuation and renovation



These data are from an actual facility, one of those involved in the North Las Vegas beryllium incident. Most of the data values resemble those from field blanks. A few show modest amounts of Be. The highest value was from a wipe collected atop an electrical box on the wall of a computer room, probably not cleaned as well or regularly as most places in the facility. Such a location is also unlikely to be a source of dermal exposure to Be in most work situations.

Distributions of Real Facility Data, continued

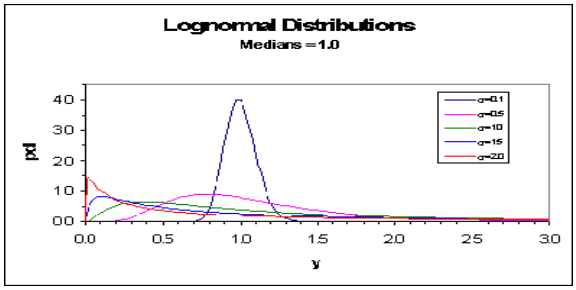
- **NTS Area 6 Shop**
  - Dusty environment; use these data for several examples



These data are from a dustier facility, a shop in Area 6 of the NTS where heavy equipment maintenance is performed. There are 62 data values; the NPUTL is the largest of these, and is well below the RC. This facility is "clean" according to the conventional criterion. The DMM fitted to these data is used extensively in subsequent slides and in The Paper for evaluating various candidate UTL procedures using smaller sample sizes.

The DMM

- **LN distribution for concentrations**
  - Shape indexed by log-scale standard deviation σ
    - Shapes from nearly normal (σ → 0) to very skewed (σ > 2)
    - σ > 2 or so is heavy-tailed, difficult to deal with
    - If σ = 2, mean = 84th percentile (think "balance point")
  - μ is log-scale mean; for given shape, e^{μ} is scale parameter



The DMM uses a LN distribution for the component representing the actual distribution of concentrations within a facility. LN distributions range from nearly normal, as the "shape parameter" σ goes to 0, to highly asymmetric (skewed) and heavy-tailed, as σ increases. The most skewed distribution in this plot has σ = 2. In informal discussions with analysts familiar with such data, σ seems to appear to be an upper limit for reasonable models for real data.

σ is the log-scale standard deviation for a LN distribution. The log-scale mean μ functions like a scale parameter.

With a symmetric distribution the mean and median (50th percentile) are the same. For a LN distribution with σ = 2, the mean equals the 84th percentile. One can think of the mean as the balance point of a distribution or histogram; the 16 percent of observations above the mean must be rather far out to balance the 84 percent below the mean.

The DMM, continued

- **Normal distributions for variation at concentration C**
  - Mean analytical response = α + ßC
  - $SD^2 = \gamma^2 + (\delta C)^2$
  - δ ≈ large-C RSD (precision)
- **Model is over-parameterized and non-identifiable**
  - Set ß = 1
- **Resulting likelihood is cumbersome**

$$f(y) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{\gamma^2+\delta^2 e^{2t}}}\,\phi\left(\frac{y-\alpha-\beta e^{t}}{\sqrt{\gamma^2+\delta^2 e^{2t}}}\right)\phi(z)\,dz$$

$t = \mu + \sigma z$, φ is standard normal likelihood

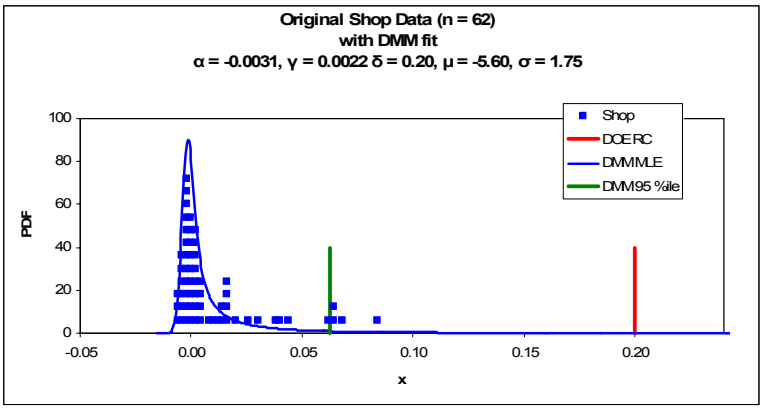
In the DMM both the mean and standard deviation depends on concentration (C). Each parameter has a physical interpretation: α is the mean response when C = 0; ß is the slope of the line relating mean response to C. γ is the standard deviation of measurements at C = 0, and δ is the precision (relative standard deviation) for large concentrations. To get the probability density function (likelihood) for an individual measurement, one multiplies the LN density function for C by the density function for measurement (y) at given C, and then integrates over the non-observable C. The result is the "unfriendly" formula given at the bottom of the slide.

One can get the same value of f(y) for different values of the six parameters; this parameterization is "non-identifiable". To solve this, we set ß = 1. This amounts to saying that, as far as one can tell from the data, the only biases in the analyses might be those around zero, but that the slope of the average instrument response is 1 in the DMM. This of course avoids all questions about digestions and calibrations.

One might similarly want to suggest setting α = 0, but doing so provides a highly significant lack of fit with the Area 6 Shop data! The Paper discusses this and similar aspects of the DMM in detail.

The DMM, continued

- **DMM fit to Area 6 Shop data**



Here is the DMM fit to the Area 6 Shop data. Of particular interest is the negative value of α; this is likely related to analytical interferences. $\sigma = 1.75$, somewhat less than the posited upper limit of 2, but still suggesting a fairly skewed LN distribution for the concentrations themselves.

Not presented in the slides, but discussed in The Paper, is the idea that one can perform likelihood ratio tests for various sub-models, such as forcing α to be 0, forcing δ to be 10 or 20 or whatever percent, and so on.

This fitting is by maximum likelihood. The likelihood function has five parameters, so one is maximizing a joint likelihood for a sample in five-dimensional space. Also, for each measurement the likelihood function is itself an integral which must be performed numerically. Hence, maximum likelihood estimation for the DMM is very computationally intensive and time-consuming, which is one reason why a "gold standard" UTL procedure using the DMM directly seems to be elusive.

The DMM, continued

- **Surprises**
  - Parameter δ never makes any difference to the fit!
    - δ (large-C precision) is a primary criterion for evaluating analytical methods!
  - With the Area 6 Shop data α is significantly negative
    - Even with blank-corrected data!
    - Fits negative interference in that dusty environment
  - With Area 6 Shop data LN3 sub-model isn't that bad
    - Significant lack of fit, but not as much as forcing α = 0
- **Other observations**
  - **Don't have UTL using DMM directly! (Research issue)**

One surprise coming from the study is that the parameter δ never makes a significant difference in the fit, so long as values of the parameters are constrained to reasonable ranges. In particular, δ is the large-C relative standard deviation. The usual concept of a Quantitation Level is a concentration above which δ is at most 10 percent; in fact, δ is perhaps THE primary metric for evaluating analytical performance of an instrument. In the maximum likelihood fitting of the DMM we require $0.05 \leq \delta \leq 0.25$; see Table 1 of The Paper.

The lack of fit induced by forcing $\alpha = 0$ was mentioned previously.

Another interesting result comes when looking at the sub-model with $\gamma = \delta = 0$; in other words, allowing a bias at $C = 0$ but not otherwise allowing a separate component of analytical variation. There is a significant lack of fit with this sub-model, but not so bad as forcing $\alpha = 0$. This sub-model is equivalent to using a three-parameter lognormal (LN3) distribution, discussed in detail here and in The Paper.

LN3 Approximate Model

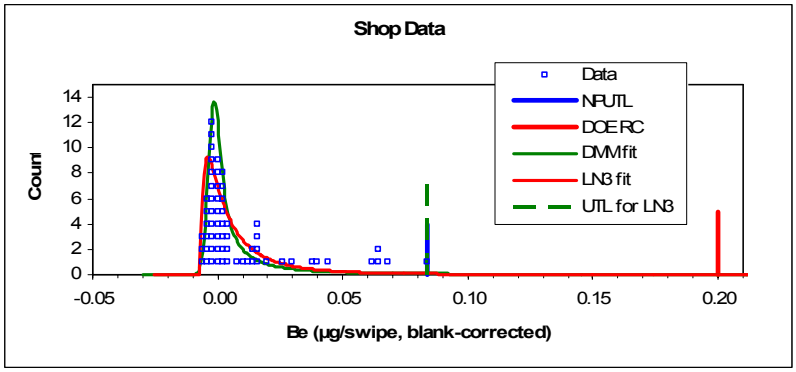
- **Procedure:**
  - Determine shift somehow [!!]
  - Develop empirical calibration factor (CF) – see Paper
  - Transform data; do normal-theory UTL on transformed scale using CF; transform UTL back to original scale
- **Cannot maintain 95% confidence over all parameter combinations**
  - Choose shift selection and CF to maintain 5% significance level reasonably over parameter combinations with 95th percentile = RC

It is possible to fit the LN3 model by maximum likelihood so long as one bounds α away from the lowest measurement. However, a more heuristic approach is adopted, as described in this slide. The idea is to add a constant (shift) to all data values to make them positive, do the LN UTL procedure, and then subtract the shift from the UTL. This is a time-honored approach to dealing with such data.

The question, though, is how to pick the shift. Since the purpose of the data analysis is to provide UTLs, which are upper confidence limits for percentiles, it would be nice if the statistical confidence would maintain the nominal value at least approximately. This is elusive however. What is more easily achieved is developing a shift selection algorithm and calibration function (CF) that keep the confidence level approximately at 95 percent at combinations of the parameter values for which the 95th percentile equals the RC. This is equivalent to protecting the integrity of the desired hypothesis test, but not worrying so much about the confidence level when the 95th percentile is rather below the RC (or above it, for that matter).

LN3 Approximate Model, continued

- **Compare LN3 and DMM fits for Area 6 Shop data (n = 62)**
  - LN3 UTL ≈ NPUTL

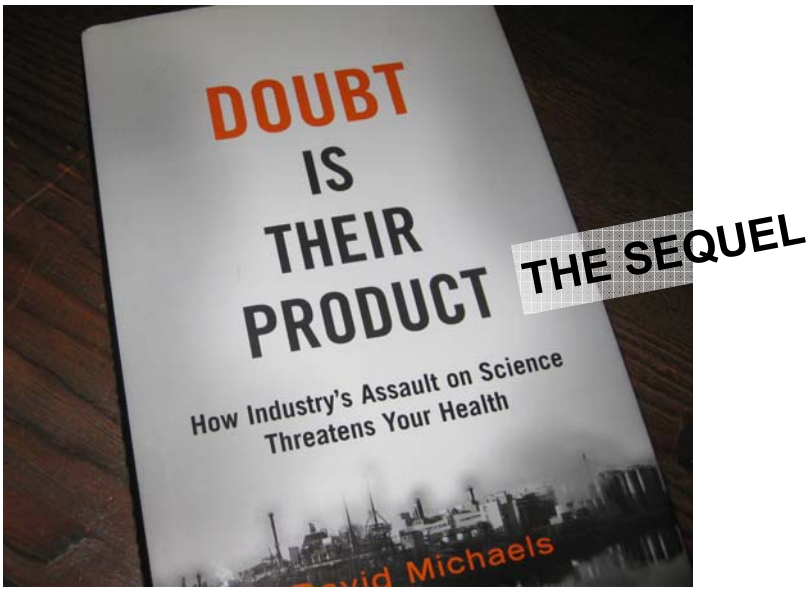


The shift selection algorithm and CF settled on are described in The Paper. The fitted LN3 distribution for the Area 6 Shop data are shown in this plot. The LN3 and DMM fits are rather similar. Of note is the fact that the LN3-based UTL is approximately the same as the NPUTL for this large data set, which agrees with our idea that nonparametric procedures and parametric procedures ought to agree with large amounts of data.

LN3 Approximate Model, continued

- ***Caveat*:**
  - As shift gets large, UTL decreases to untransformed normal-theory UTL, but expected significance level becomes less reliable
  - Significance level integrity is quite sensitive to shift selection algorithm
  - **Presents opportunity for twisting data to provide desired conclusion!**

There is a concern, though, in that as the shift increases UTL decreases. For very large shifts, the UTL decreases to what it would be if one just used normal-theory UTLs ignoring the skewness of the data. The significance level (chance of deciding "clean" when the 95th percentile is above the RC) will increase as a result. This presents opportunity to "fudge the data".



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Statisticians who "sophisticate" data to provide desired conclusions run the risk of eventual exposure.

Censored-Data LN (LN2) Methods

- **See Davis (2006) for comparison of censored-data LN2 methods**
  - For truly LN2 data, LN2 cMLE with calibration factor beats other methods tested
  - Procedure reliable up to around 70% or so "nondetects"
  - But how does it work with data from a DMM?

The third major portion of The Paper addresses the performance of censored-data LN2 methods. A particular procedure based on maximum likelihood estimation was the "winner" in a study conducted and presented in 2006, providing reliable confidence for reasonable parameter values with up to around 70 percent less-than data values so long as the distribution is actually LN. But the current question revolves about how this procedure performs with DMM data.

Censored-Data LN (LN2) Methods, continued

- **Example: repeated samples from DMM for Area 6 Shop data,**
  - n = 15 and n = 25
- **Excerpt from Table 9 of Paper follows**
  - Four different RLs: 0.05, 0.02, 0.01, 0.002
  - Top line: use all samples regardless of % nondetects
  - Bottom line: use only samples with at most 70% nondetects
  - "Uses" column is proportion of samples used
  - "Conf" column is proportion of samples with UTL ≥ 95th %-ile
  - "Power" column is proportion of samples with UTL ≤ RC
    - **= chance of deciding "clean"**

A brief excerpt of the results appears on the next slide. Four different RLs are used: 0.05 (one-fourth of the RC), 0.02 (one-tenth), 0.01 (one-twentieth), and 0.002 (one-hundredth). Two approaches were used. The top line for each RL gives the results with all samples used, regardless of the proportion of less-thans in the sample, whereas the bottom line for each RL uses only the samples with at least 30 percent detects.

This particular excerpt again uses the DMM fitted to the Area 6 Shop data. This facility is clean, and so the major interest is in the column "Power", which is the proportion of samples coming to the correct conclusion.

This particular excerpt uses sample sizes n = 15 and n = 25. The Paper also shows results for n = 45.

Censored-Data LN (LN2) Methods, continued

Simulation Results (using cMLE UTL for LN2 using CF)

←RL decreasing

	Area 6 Shop, n = 15			Area 6 Shop, n = 25		
	uses	Conf.	Power	uses	Conf.	Power
LN3	all	90.3%	47.9%	all	86.1%	66.0%
LN2 RL = 0.05	all	57.8%	61.1%	all	76.6%	57.6%
"	0%	x	x	0%	x	x
LN2 RL = 0.02	all	77.2%	45.2%	all	86.0%	50.2%
"	6%	96.7%	15.0%	3%	100.0%	6.7%
LN2 RL = 0.01	all	87.1%	36.5%	all	90.6%	42.5%
"	28%	96.4%	22.4%	23%	97.0%	28.9%
LN2 RL = 0.002	all	97.4%	17.1%	all	97.3%	24.6%
"	88%	97.8%	16.7%	94%	97.7%	24.0%

The yellow highlights show what proportion of samples have at least 30 percent detects. If one uses only those samples, the power is quite low compared with that of the LN3 UTL procedure. The only RL for which one consistently has at least 30 percent detects is the lowest, 0.002. However, power is again low with that RL regardless of whether one includes all samples or not.

The cMLE UTL approach can be carried out even if there is only one observation above the RL (if there are none, set UTL = RL). When all samples are included regardless of the proportion of less-thans, the power is better, particularly for RL = 0.05 and 0.02.

Censored-Data LN (LN2) Methods, continued

Simulation Results (using cMLE UTL for LN2 using CF)

	Area 6 Shop, n = 15			Area 6 Shop, n = 25		
	uses	Conf.	Power	uses	Conf.	Power
LN3	all	90.3%	47.9%	all	86.1%	66.0%
LN2 RL = 0.05	all	57.8%	61.1%	all	76.6%	57.6%
"	0%	x	x	0%	x	x
LN2 RL = 0.02	all	77.2%	45.2%	all	86.0%	50.2%
"	6%	96.7%	15.0%	3%	100.0%	6.7%
LN2 RL = 0.01	all	87.1%	36.5%	all	90.6%	42.5%
"	28%	96.4%	22.4%	23%	97.0%	28.9%
LN2 RL = 0.002	all	97.4%	17.1%	all	97.3%	24.6%
"	88%	97.8%	16.7%	94%	97.7%	24.0%

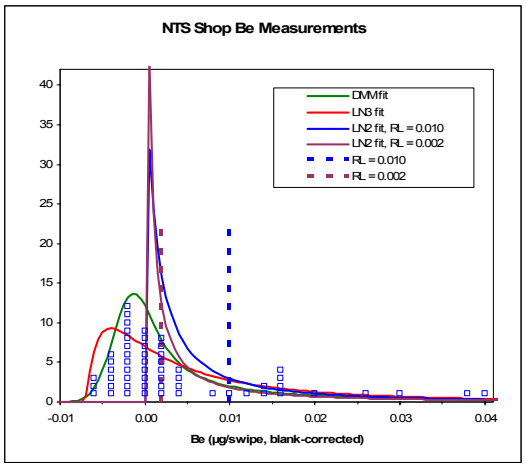
These are the procedures that have power roughly comparable to that of the LN3 procedure with data like those of the Area 6 Shop, but smaller sample sizes. The message here, and elsewhere in The Paper, is that one should not make RL too low.

It should be mentioned that this procedure with RL = 0.05 does have a somewhat high significance level with DMMs having 95th percentile = RC where $\sigma = 2$. Accordingly, there are suggestions in The Paper that RL should not be made too high either, at least if one anticipates obtaining censored data and using these sorts of UTL procedures.

An interest follow-on research challenge would be to quantify recommendations for setting RLs using these sorts of considerations. This would be a challenge even having copious data from a particular lab. Setting Data Quality Objectives (DQOs) or Measurement Quality Objectives (MQOs) based on these considerations for a new laboratory to be brought into a study would likely present further difficulties.

Censored-Data LN (LN2) Methods, continued

- **Why does lowering RL backfire?**



Why does lowering RL backfire? The mechanism seems to be that any censored-data method based on an assumed LN distribution implicitly forces all the negative values into the interval (0, RL). So with RL = 0.01, the negative are implicitly considered has having come from the interval between zero and the right vertical dotted line, and with RL = 0.002 they are implicitly considered as having come from the interval between zero and the left vertical dotted line. The implicitly fitted LN distributions thereby end up being much more skewed than the actual data, increasing the UTL and thereby reducing the power.

Censored-Data LN (LN2) Methods, continued

- **Research and political need**
  - **Take considerations like these into account when determining project-specific RLs!**
  - Currently, RLs are determined from point-of-view of interpreting single measurements only

Quasi-Nonparametric UTL procedure

- **NPUTL is largest value or RL <u>regardless of how far below RC</u>**
  - How to get credit for that distance?
  - Ed's brainstorm: use LN assumption, but ONLY to bound percentile of NPUTL
- **For example, with n = 15, largest data value is 95% confidence NPUTL for 81.9th percentile**
  - If we <u>are willing to assume</u> LN $\sigma \leq 2$, ratio between 81.9th and 95th percentiles is at most 0.23, so if NPUTL is at most 0.23 * RC, decide "clean"
  - Would require RL at most 0.23 * RC, of course

Ed Frome of ORNL has suggested an approach based on assuming an LN distribution for the data, but making rather minimal use of that. The idea is that if $n < 59$ the largest data value is a NPUTL with 95 percent confidence, but for a lower percentile than the 95th percentile. But if the RL or largest data value is less than the RC AND one can use a minimal LN assumption to get a handle on the ratio of the percentiles, one might nonetheless be have a reasonable procedure comparing the largest data value with an appropriate fraction of the RC. The slide gives the details for $n = 15$; the percentile is the 81.9th percentile, and the ratio is 0.23.

The ratio between percentiles again requires an upper bound on the LN parameter σ; the ratio 0.23 comes from assuming $\sigma \leq 2$.

Quasi-Nonparametric UTL procedure, continued

- **Evaluation**
  - Significance levels comparable to nominal 5% so long as bound on σ is reasonably tight
  - Power is generally lower than LN3 procedure
  - Power is generally comparable with cMLE LN2 procedure with RL is not too low and nondetect fraction ignored
  - **DOES AVOID CONTROVERSIAL LN3 REQUIREMENT FOR UNCENSORED DATA!**
  - **Would need to take into account in setting RL, though**

As discussed in The Paper, this procedure seems to work reasonably with DMM data, and the sensitivity to σ may actually be nicer than with the cMLE LN2 UTL procedure. Power is generally lower than that of the complete-data LN3 procedure.

This procedure avoids the controversial need for obtaining uncensored data, but only so long as the RL and largest data value are low enough. This again places requirements on setting the RL that go beyond current practice.

Summary

- **Have decent model (DMM) for ICP-AES and ICP-MS data**
  - More complicated than conventional lognormal (LN2) model
  - Can fit via MLE to uncensored data
  - Handles negative values appropriately
- **Have an approximate UTL procedure based on shifted LN (LN3) distribution for uncensored data**
  - Protects desired significance level reasonably so long as σ is not too large
  - **σ > 2 is problematic for any procedure except NPUTL (n ≥ 59)**
  - **Requires uncensored data**

Here's the summary.

Summary, continued

- **With data resembling DMM data**
  - By censoring data one loses information, making "clean" decision more elusive
  - Optimal censored LN2 method does NOT work as well with actual data!
    - Sensitive to RL choice, too conservative if RL too small!
    - Recommend trusting model and ignoring % nondetects in sample!
  - Quasi-nonparametric LN procedure is promising, but does require (a) LN assumption with (b) reasonably tight upper bound on LN σ
- **Send request for Paper to**

charles.davis@envirostat-nv.com

The Paper will also be posted on the portion of the BHSC website for the North Las Vegas November 2009 meeting.